

Mergesort

Mergesort

- ▶ Mergesort (divide-and-conquer)
 - Divide array into two halves.

A	L	G	O	R	I	T	H	M	S
---	---	---	---	---	---	---	---	---	---

A	L	G	O	R
---	---	---	---	---

I	T	H	M	S
---	---	---	---	---

divide

Mergesort

- ▶ Mergesort (divide-and-conquer)
 - Divide array into two halves.
 - Recursively sort each half.

A	L	G	O	R	I	T	H	M	S
---	---	---	---	---	---	---	---	---	---

A	L	G	O	R
---	---	---	---	---

I	T	H	M	S
---	---	---	---	---

divide

A	G	L	O	R
---	---	---	---	---

H	I	M	S	T
---	---	---	---	---

sort

Mergesort

- ▶ Mergesort (divide-and-conquer)
 - Divide array into two halves.
 - Recursively sort each half.
 - Merge two halves to make sorted whole.

A	L	G	O	R	I	T	H	M	S
---	---	---	---	---	---	---	---	---	---

A	L	G	O	R
---	---	---	---	---

I	T	H	M	S
---	---	---	---	---

divide

A	G	L	O	R
---	---	---	---	---

H	I	M	S	T
---	---	---	---	---

sort

A	G	H	I	L	M	O	R	S	T
---	---	---	---	---	---	---	---	---	---

merge

Merging

► Merge.

- Keep track of smallest element in each sorted half.
- Insert smallest of two elements into auxiliary array.
- Repeat until done.

smallest



A	G	L	O	R
---	---	---	---	---

smallest



H	I	M	S	T
---	---	---	---	---

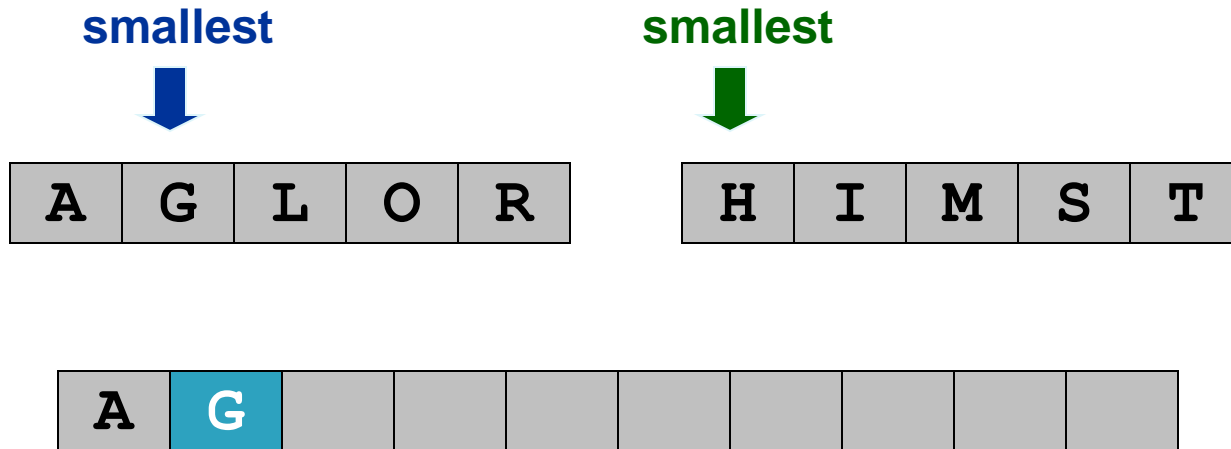
A									
---	--	--	--	--	--	--	--	--	--

auxiliary array

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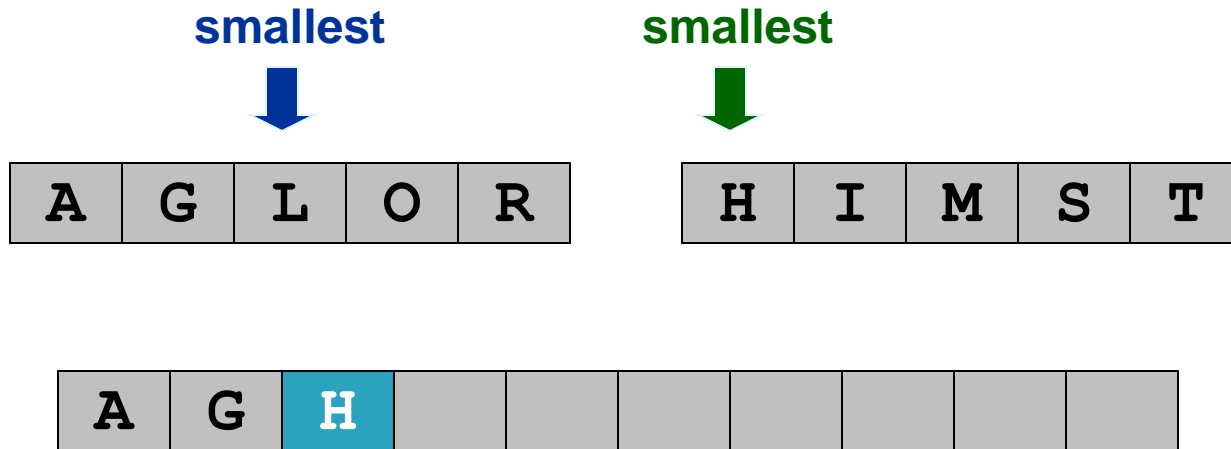


auxiliary array

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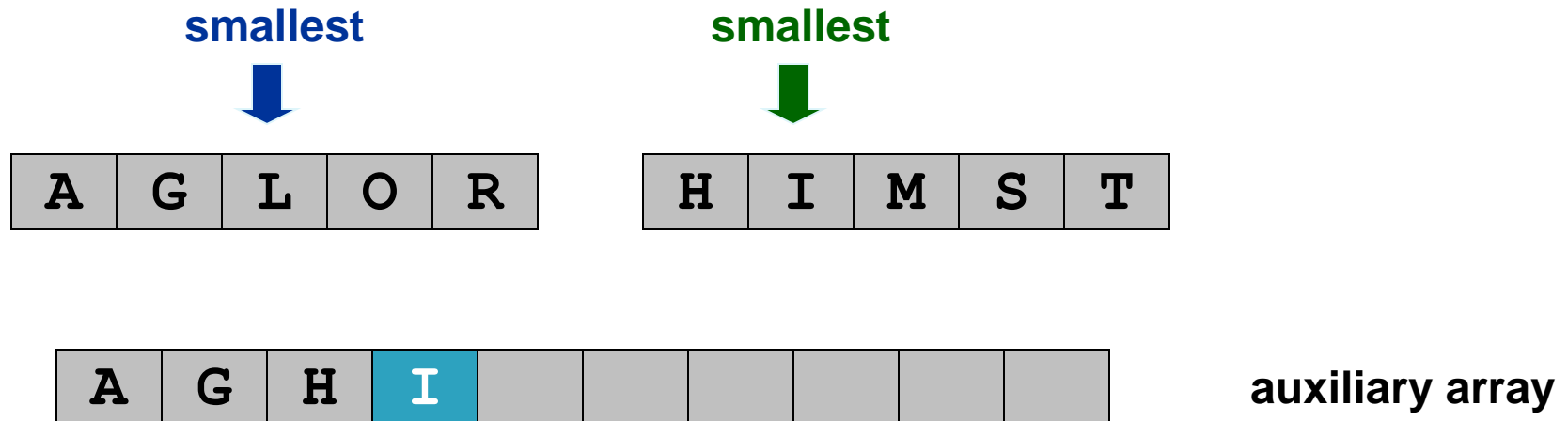


auxiliary array

Merging

► Merge.

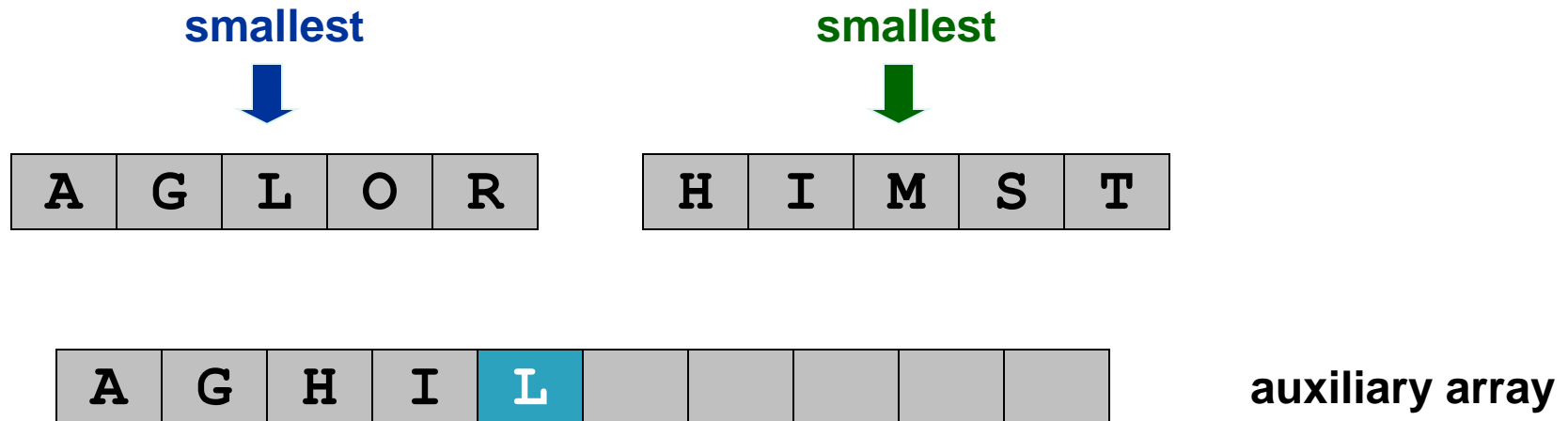
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Merging

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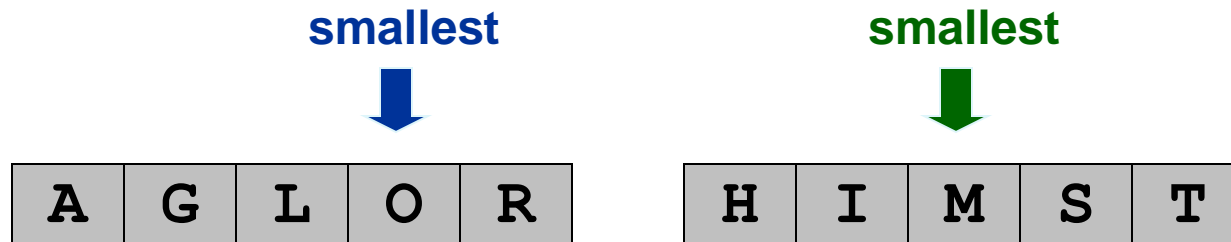
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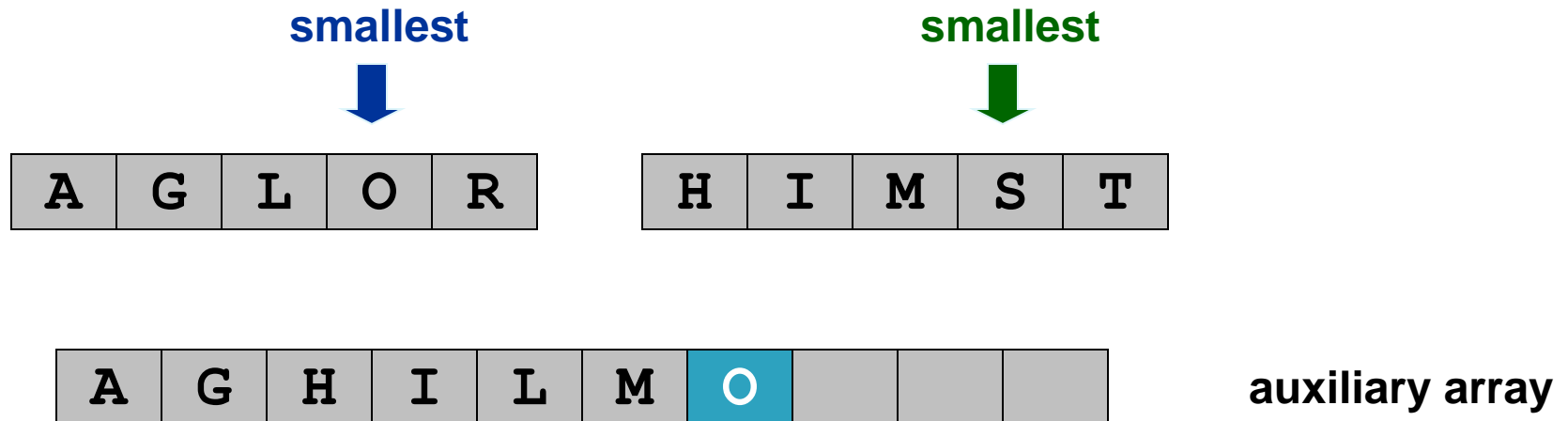


auxiliary array

Merging

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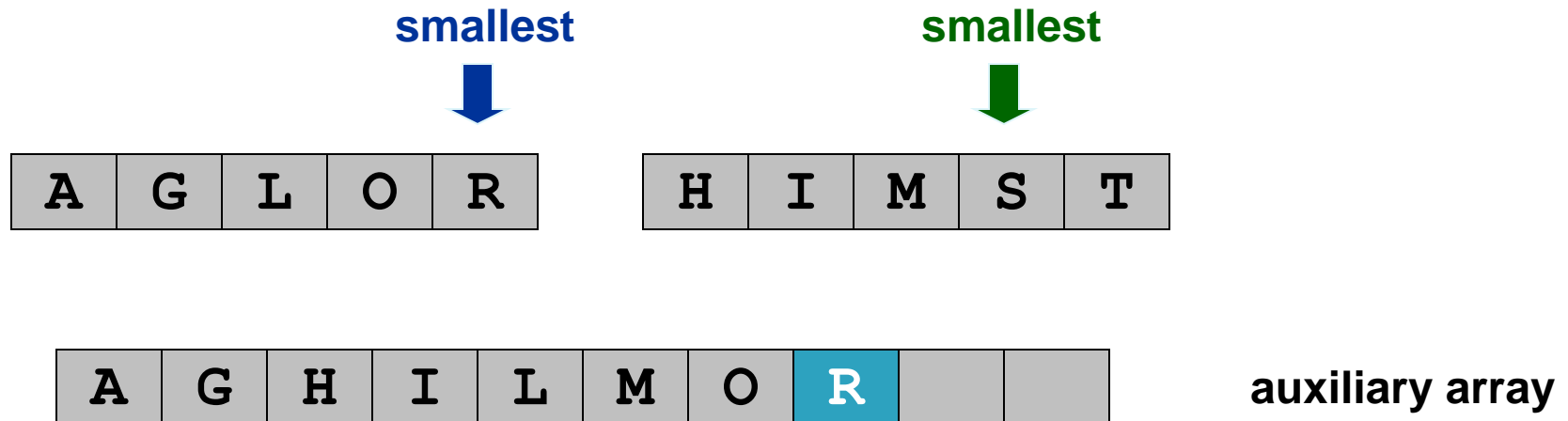
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Merging

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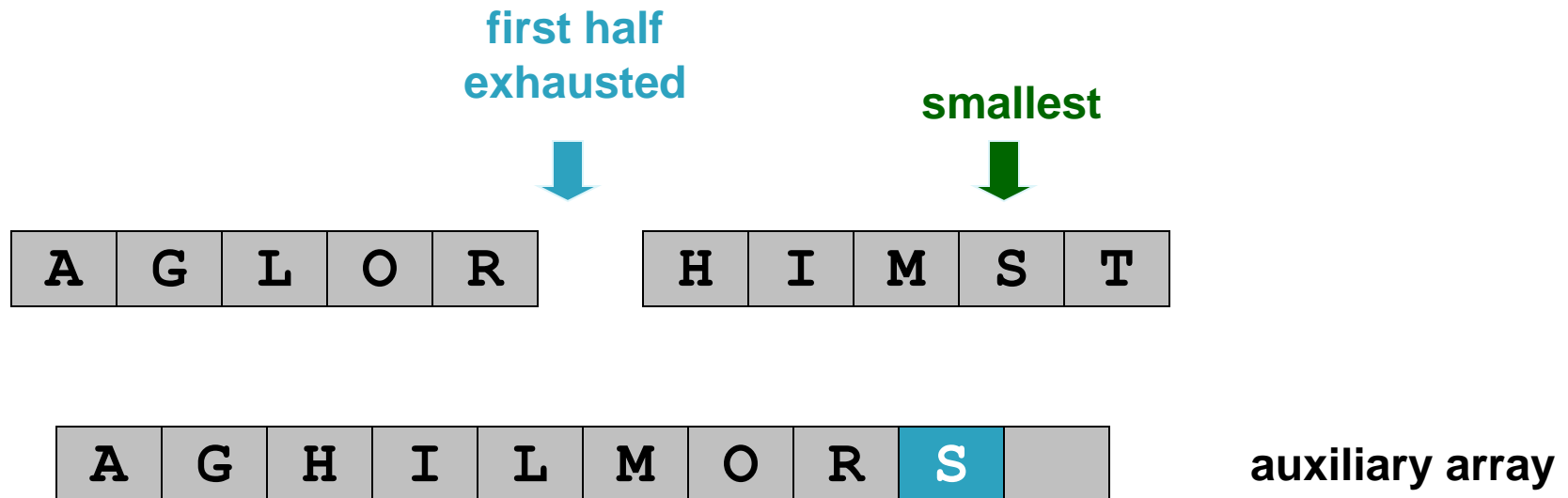
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Merging

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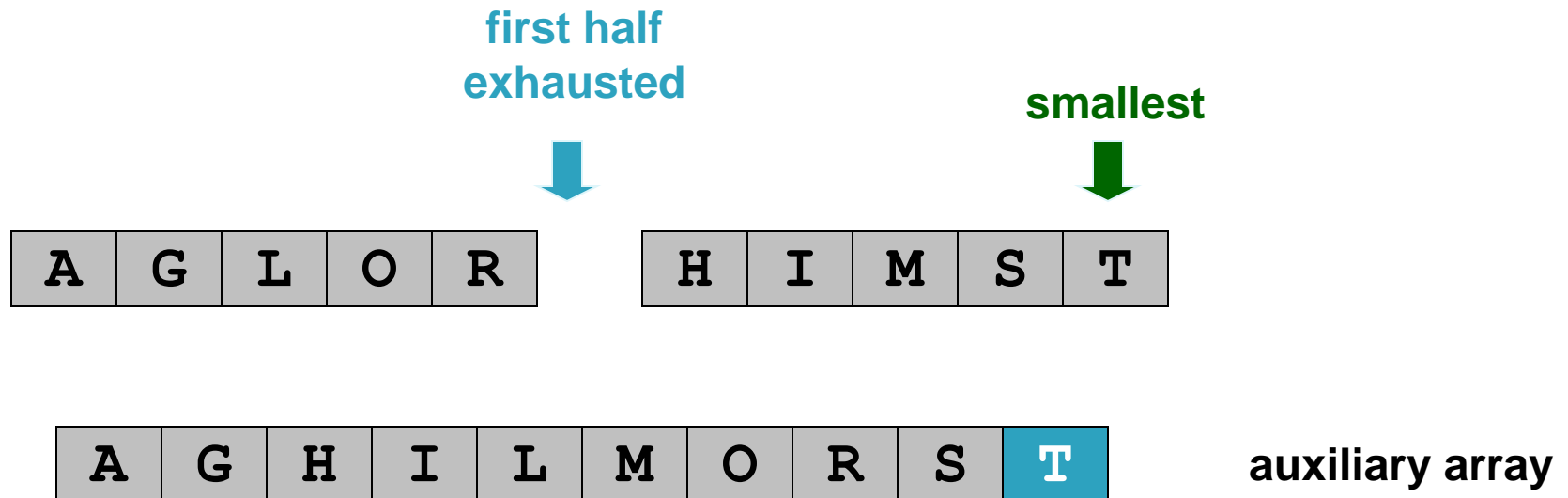
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Merging

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Implementing Mergesort

mergesort (see Sedgewick Program 8.3)

Item aux[MAXN];

← uses scratch array

```
void mergesort(Item a[], int left, int right) {  
    int mid = (right + left) / 2;  
    if (right <= left)  
        return;  
    mergesort(a, left, mid);  
    mergesort(a, mid + 1, right);  
    merge(a, left, mid, right);  
}
```

Implementing Merge (Idea 0)

```
mergeAB(Item c[], Item a[], int N, Item b[], int M )
{ int i, j, k;
  for (i = 0, j = 0, k = 0; k < N+M; k++)
  {
    if (i == N) { c[k] = b[j++]; continue; }
    if (j == M) { c[k] = a[i++]; continue; }
    c[k] = (less(a[i], b[j])) ? a[i++] : b[j++];
  }
}
```


Implementing Mergesort

merge (see Sedgewick Program 8.2)


```
void merge(Item a[], int left, int mid, int right) {
    int i, j, k;

    for (i = mid+1; i > left; i--)
        aux[i-1] = a[i-1];
    for (j = mid; j < right; j++)
        aux[right+mid-j] = a[j+1];

    for (k = left; k <= right; k++)
        if (ITEMless(aux[i], aux[j]))
            a[k] = aux[i++];
        else
            a[k] = aux[j--];
}
```



copy to
temporary array



merge two sorted
sequences

Mergesort Demo

- ▶ Mergesort The auxiliary array used in the merging operation is shown to the right of the array $a[]$, going from $(N+1, 1)$ to $(2N, 2N)$.
- ▶ The demo is a dynamic representation of the algorithm in action, sorting an array a containing a permutation of the integers 1 through N . For each i , the array element $a[i]$ is depicted as a black dot plotted at position $(i, a[i])$. Thus, the end result of each sort is a diagonal of black dots going from $(1, 1)$ at the bottom left to (N, N) at the top right. Each time an element is moved, a green dot is left at its old position. Thus the moving black dots give a dynamic representation of the progress of the sort and the green dots give a history of the data-movement cost.